Improvement of Membership Function Identification Method in Usability and Precision

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Keywords: Trapezoidal membership function, Computer-aided identification of membership function, Boundary asymptotic estimation method, Computerized fuzzy graphic rating scale method

Abstract

Fuzzy sets are used in various fields. For those who have little knowledge on fuzzy set theory we find that improving usability of membership function identification method is really important. This paper aims to propose this new method improved in usability and precision, and then to verify the method through experiments. First, we propose the method to identify a trapezoidal membership function, BASE method (boundary asymptotic estimation method). The features of this method are ternary rating of membership grades, asymptotic estimation of boundaries of 1-level set and support set, and effective and recursive selection of elements using computers. Results from psychological experiments are showing that the BASE method is superior to the computerized fuzzy graphic rating scale both in usability for inexperienced users of fuzzy set theory and precision.

1. Introduction

Wrong interpretation of settling membership functions: As already known, we cannot define unique membership function (MF in short) for each fuzzy concept because of vagueness and diversity of data from individual differences. Many researchers therefore consider that they can settle the functions arbitrarily without adequate procedures. Some adequate procedures however must be applied to handle individual differences of their shapes, such as in subjective evaluation [1].

Previous works and their problems: Due to the above necessity several researchers proposed some identification methods and verified them through experiments [2-8]. Most of the methods attach weight to precision of the identified MFs, namely, ability to describe the shapes. However, as fuzzy sets are used in various fields, improving usability of the methods is really important for those who have little knowledge on fuzzy set theory. In above sense the methods proposed in the past have rooms for improvement.

Aims of this work: Aims of this paper are to propose a usable identification method for a trapezoidal MF and to validate the method experimentally. First, we propose a new MF identification method (MFIM in short) with considering its usability for "*novice*" users of fuzzy set theory in the section 2. Next in the section 3 three different user interfaces of the proposed method, BASE, are evaluated upon its implementation onto computers. Then in the section 4 the proposed method is compared with the computerized fuzzy graphic rating scale [8] both in usability and precision aspects. In this paper we deal with identification methods for a trapezoidal MF, i.e., unimodal distribution, since most fuzzy concepts and fuzzy numbers used in applications are unimodal ones and even the identification methods for unimodal ones are not well developed.

2. Theoretical Explanations of BASE Method

2.1 Aims of This Section

In this section, we first review two major previous MFIMs as background of this work and describe problems concerning usability. Then we propose a new MFIM, BASE method, and explain its algorithm and the features.

2.2 Previous MFIMs and Some Problems

Categorization of MFIMs: Many researchers have already proposed many kinds of MFIMs, and some researchers have compared those methods through experiments in aspect of the obtained MF [2-8]. **Figure 2.1** shows a categorization of major MFIMs by Nakamura [9]. The methods are divided into two major categories. One category includes methods that a subject rates membership grades for elements presented by either experimenters or systems. We call this type of the methods RM: rating membership grades for presented elements. The other includes methods that a subject answers elements or intervals that correspond to membership grade(s) presented by either experimenters or systems. We name this type of the methods AE: answering elements corresponding to membership grade(s). As seen from the figure 2.1, the RM has a remarkable variety of estimation of membership grades. In the variations, DR: direct rating is often used.

Schema of two popular methods: Both the DR and the AE are popular methods for identifying MFs in some real applications. As preparation of our new method proposed later, we explain their strategies and problems that spoil their usability. **Figure 2.2** shows schemes of how a MF is identified using the DR and the AE.

In the DR the discrete elements to be rate are chosen in an equal interval. A subject or responder rates to what extent the elements belong to the fuzzy concept to be identified. The subject imagines only partial shape of the MF around the presented element through "*scope for rating*." However, there are some problems to lower usability. First, the method forces subjects to describe the membership grade precisely and rigidly. Second, the more

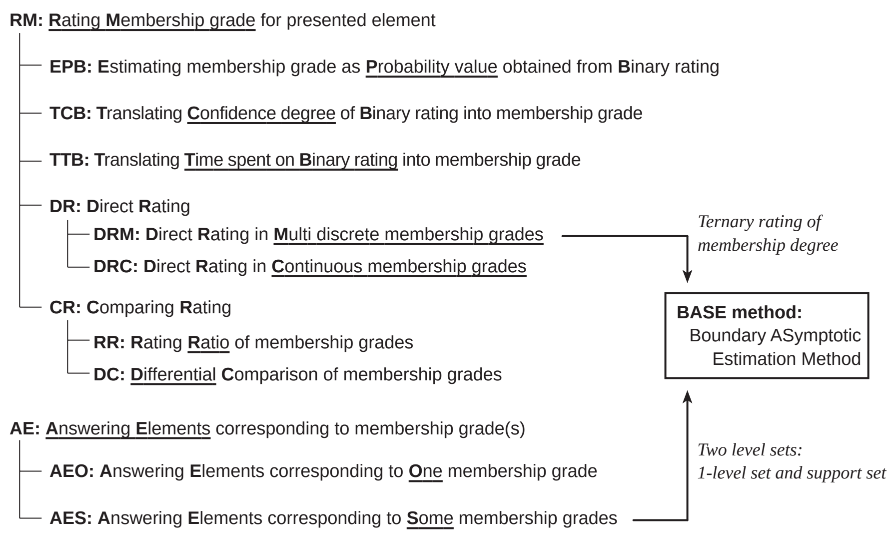
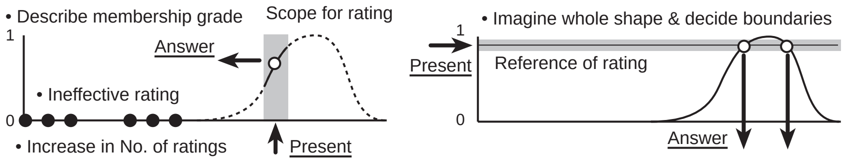


Figure 2.1. Categorization of MFIMs based on ways of subjective judgments and position of the BASE method in the categories.



(a) The DR (b) The AE

Figure 2.2. Schemes of two major methods: DR and AE, and their problems

precise MF we obtain, the more elements the subjects have to rate. Third, it forces them to rate ineffective elements not necessary to, if their rating values were well deduced from the values of other elements.

In the AE a subject answers a set of elements or an interval that corresponds to a given membership grade. An advantage of this method is that it can complete the identification by answering only small number of boundaries. There are problems in this method however that spoil its usability. Namely, the subject is forced to imagine the whole shape of the fuzzy set before identification and to decide the boundaries directly comparing the fuzzy set with the presented membership grade as "*reference of rating*."

2.3 Principle of BASE Method

Algorithm of the BASE method: As mentioned above, both of the two popular methods often used have some problems that lower their usability. To reduce some of the problems, we propose a new method, named BASE method: *boundary asymptotic estimation method* [10]. This method estimates *indirectly* two level sets for *a trapezoidal MF*, i.e., a unity level set and a support set. That is, the obtained fuzzy set is comparable to one obtained from the AE. This method adopts *ternary rating* of the membership grade for the element selected *effectively and recursively*.

Ternary rating: Figure 2.3 shows an algorithm of the BASE method and illustrates its process of identification of a MF. As shown in the upper left figure, a user of this method rates a membership degree of the element presented by the system at ternary values: "*the element belongs to the fuzzy set*," "*the element does not belong to it*," and "*the element neither belongs nor does not belong to it*." We can think here two thresholds of the rating: λ_1, λ_0, corresponding to "*belong*" and "*not belong*" respectively. We can approximate λ_1, λ_0 to the ideal thresholds, *unity* and *zero*, by instructing the user to rate at "*neither*" whenever she/he gets confused to rate at either "*belong*" or "*not belong.*"

Asymptotic estimation of boundaries: The right figure shows a flow of identification. The process is divided into three stages: "*search in binary tree*," "*estimate 1-level set*," and "*estimate support set*" in order of progress. The purpose of the first stage is to search only one element rated at "*belong*." At this stage the elements presented for the user are selected based on a unique binary tree. The user rates the element at the ternary values repetitively until the element rated at "*belong*" is found. In other words, the system recursively presents a center element of two elements that have already been rated. In the middle left figure, the elements numbered from 1 to 5 belong to this stage.

Another stages aim to narrow width of the intervals that contain the boundaries of the two level sets. At the second stage the two boundaries of the 1-level set are estimated. As shown in the middle left figure, the system presents the center element of the interval that has the element rated at "*belong*" at only one end, and the user rates the element. These two states are repeated until all the upcoming intervals that should be rated become smaller than the given minimal width (the elements numbered from 6 to 11). At the last stage the

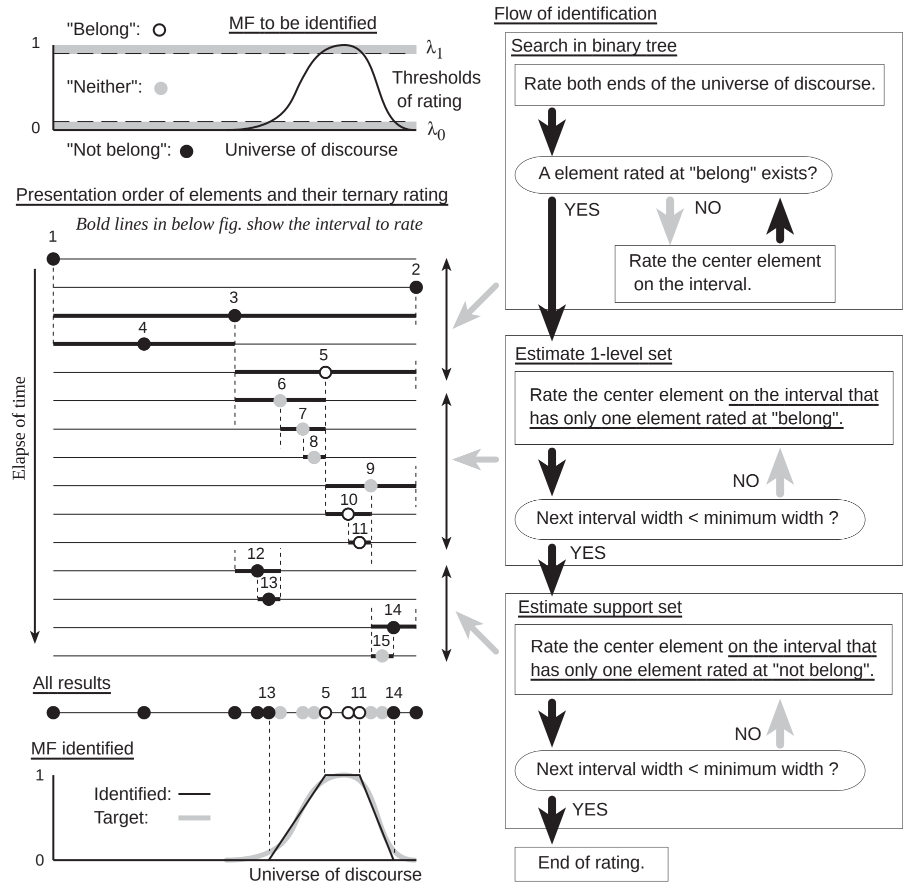


Figure 2.3. A schema of how a MF is identified by the BASE method.

two boundaries of the support set are estimated. The same way as the second stage is used except for process that the anchor element is changed from "*belong*" into "*not belong*." In the figure the elements numbered from 12 to 15 belong to this stage.

Finally, as shown in the bottom left figure, the trapezoidal fuzzy set is constructed. The lower and upper elements rated at "*belong*," i.e., the element of 5 and 11 in the figure, are assigned to the lower and upper boundaries of the 1-level set respectively. Meanwhile the upper element rated at "*not belong*" in all the elements located in the left side of the lower boundary of the 1-level set, i.e., the element of 13 is assigned to the lower boundary of the support set. Also the lower element rated at "*not belong*" in all the elements located in right side of the upper boundary of the 1-level set, i.e., the element of 14 is assigned to the upper boundary of the support set.

Verification of the algorithm through computer simulation: We executed a computer simulation experiment to confirm that the algorithm works completely. We divided the universe of discourse into 32 intervals. 58905 possible trapezoidal and triangular MFs were constructed from either two, three or four of arbitrary different elements in the 33 elements. Then we checked whether or not all the MFs are identified by the proposed algorithm. As the results, no error is detected. So it confirms that this algorithm works to estimate an arbitrary normal convex fuzzy set as a trapezoidal MF completely. The result also shows expected mean value and SD of the number of ratings per fuzzy set are 14.6 and 2.5 under the above condition.

Position of the BASE method: As mentioned above, the BASE method equips both of the features of the DR and the AE. Therefore, there is no category that includes the BASE method exactly. As drawn in figure 2.1, it locates between the DRM: direct rating in multi discrete membership grades and the AES: answering elements corresponding to some membership grades.

Good features of the BASE method: We can reduce and improve some of the problems concerning usability of the DR and the AE using the BASE method. The problem of precise rating of a membership grade in the DR is reduced since the BASE method requires only the ternary rating. Furthermore the BASE method improves the other problem of increasing the number of rating, as seen from the results of averaged number of rating obtained from the simulation. Meanwhile the problems in the AE that user is forced to imagine the whole shape prior to identification and to answer the boundaries directly disappear since the BASE method adopts the ternary rating and estimates these boundaries indirectly. Above all, an improvement in usability is exemplified in the section 4 comparing with the AES through psychological experiments.

3. Evaluation of User Interfaces of BASE Method thru Experiment

3.1 Aims of This Section

In this section, we first evaluate three different user interfaces of the BASE method through experiment from a viewpoint of usability to implement it on computers. Then we examine availability of answering time per element for improving MFs identified in precision.

3.2 Experiment Procedures

Stimuli: We adopted twelve verbal expressions of tallness, which consists of a verbal hedge and "tall" such as "very tall," as fuzzy concept to be identified. The stimuli were divided into three groups, and the groups were assigned to three different feed back methods explained below. The first stimulus in each group is a dummy one. So these three stimuli are not used in analysis.

Systems: Figure 3.1 shows a user interface of the BASE method. In this experiment, we used Macintosh computer with a 13 *inches* display. The interface consists of a fuzzy concept to be identified, a graphic scale with a moving pointer, a feed back area of histories of rating, and four buttons for answering. The graphic rating scale has two verbal anchors and three ticks. It is 256 *pixels* in length, corresponding to around 95 *mm* on the 13 *inches* display. The left and right extremities of the scale mean "*absolutely short*" and "*absolutely tall*" respectively in this experiment. The pointer located under the scale indicates a current element to be rated. We adopted eight *pixels* as the minimal interval. The subjects are asked to rate membership degree of the presented element as an extent that two degrees of tallness between the element and the fuzzy concept agree with. So we adopted {"*agree*", "*not agree*", "*neither*"} as the ternary rating values instead of {"*belong*", "*not belong*", "*neither*"} mentioned in the section 2.

Expressing histories of rating: To examine influence of expressing histories of rating on its usability, we compared three different feed back methods. One is that neither positions of elements nor their rating values are displayed. Another one is that only the positions are displayed in feed back area. Last one is that both the positions and their values are dis-

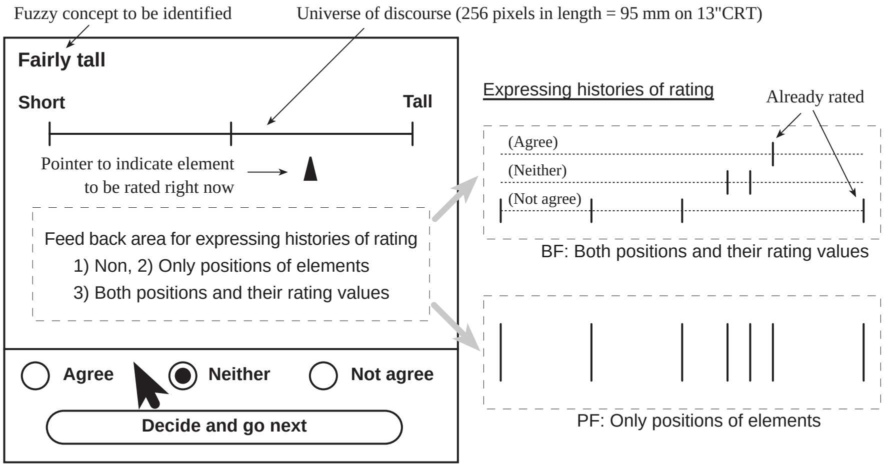


Figure 3.1. Examples of the BASE method implemented on computer and illustrations of different feed back methods.

played in the area. We call these conditions "*NF*," "*PF*" and "*BF*" in order.

Measuring items: We measured answering time per element in a unit of 1/60 *s* as well as the ternary value for each element. Moreover, we asked the subjects to rank their usabilities between the three different feed back methods using the pair comparison method.

Subjects: 36 subjects participated in this experiment as volunteer. They were graduate students and undergraduates at Kyoto Institute of Technology. All the subjects had little knowledge on fuzzy set theory. All of them were native Japanese speakers. Thus all materials for the experiment were displayed in Japanese. These subjects were divided into six groups. The groups were assigned to different order of the three feed back methods to counterbalance influence of experiment order on the answering time.

3.3 Results and Discussions

3.3.1 Feed Back Method Used in The BASE Method

Subjective evaluation of the three feed back methods: Figure 3.2 shows the results obtained from the pair comparison of their usability between the different feed back methods. As seen from these figures, BF is significantly usable than NF and PF at significance level of 5 and 0.1 % in the pair test, where the frequencies rated at "*even*" are not used in this test. Meanwhile, there is no significant difference of the frequencies between NF and PF.

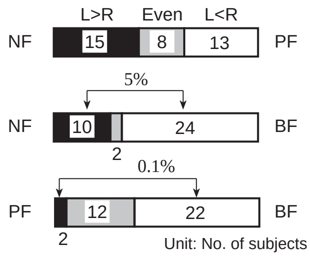


Figure 3.2. Results of pair comparison of usability between the different feed back methods.

Objective evaluation of the three methods: Figure 3.3 shows examples of identified MFs and results of their answering time. We then averaged the number of ratings per hedge, answering time per hedge, and width of the 1-level set and the support set for each feed back method. **Figure 3.4** reveals the mean values and their SDs for the three feed back methods along with the results from the t-test of difference between two mean values in two-tail. As seen from the figures, there is no significant difference among three different feed back methods except for the pair of NF

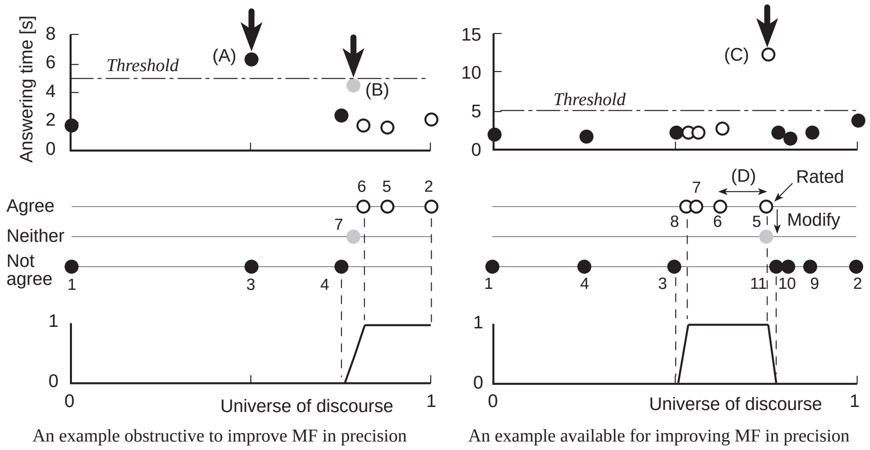


Figure 3.3. Examples of obtained results and availability of answering time for improvement in precision of MF identified

and BF in the averaged answering time and the pair of NF and PF in the width of the support set at significance level of 5, 10% respectively.

Recommended feed back method: In summary the obtained results of objective evaluation reveal that the differences of the feed back methods do not influence the objective values, i.e., the MFs. Contrarily the results of subjective evaluation clearly show that the subjects prefer "*expressing positions of elements and their ratings*" to the other feed back methods. Consequently, we recommend expressing both the elements and the ratings as the feed back method of the BASE method based on the subjective evaluation.

3.3.2 Availability of Answering Time toward Improvement of MF in Precision

Problem of forced binary rating in the BASE method: One of the features of the BASE method is that the element hard to rate at either "*belong*" or "*not belong*" is simply rated at "*neither.*" This feature is useful for improving not only in usability, but also in precision, mentioned in 2.3. However, some users prefer forced binary rating to ternary rating. The forced binary rating causes two thresholds: λ_1, λ_0, to bring near 0.5 in figure 2.3. So it decreases precision of the MF identified. If an element that is forcibly rated at "*belong*" or "*not belong*" would be detected on identification anyhow, we could improve the MF in precision by changing the rating into adequate one, i.e., "*neither.*" Answering time is known as a measure of confusion of rating. Oda has already tried to estimate membership degree based on it [5]. So we examine availability of answering time per element.

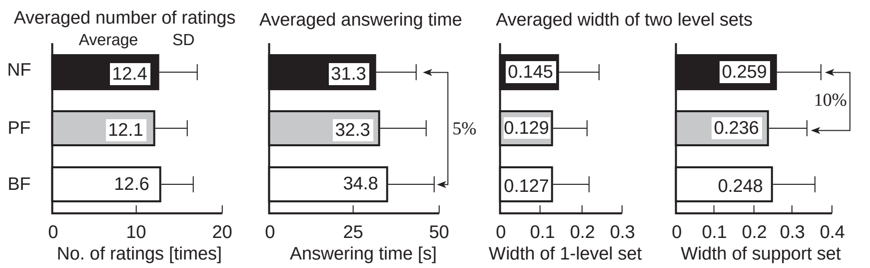


Figure 3.4. Comparison of the obtained results among three different feed back methods.

Availability of answering time: If we could use the answering time for improving, the three conditions must be satisfied. First, there is an element that has remarkably long delay of the answering time. Second, the delayed element is at an end of level sets. Third, the element is detected on identification. We check these conditions from the measured answering time in order.

For the first condition, there are the elements that have longer answering time than the others, marked with an arrow in the figure 3.3. However the second condition is not satisfied. For example, the element of (A) in the left figure belongs to "*not agree*" obviously, but it takes long time to be rated. These kinds of delay are caused by the other sources, such as lowness of arousal level, but not confidence of rating. Contrary like as the element of (C), the answering time could be used for improving. Finally for the third condition we could adequately discriminate these long delayed answers *after the identification*. However, it is hard to settle the threshold for detection on identification, since a range of answering time cannot be estimated prior. From these results, we conclude as follows. It is difficult to improve the MF identified in precision by changing the subject's rating values based on detecting long delay of the answering time for each element on identification. However, the long delay is effective to check the inadequate rating, i.e., forced binary rating with confusion, after end of all the identification.

4. Comparative Studies on BASE Method

4.1 Aims of This Section

In this section, we compare the BASE method with the computerized fuzzy graphic rating scale (FGRS) [8], which is one of the popular methods often used in the applications. Then we evaluate usability of these two methods for "*novice*" users of fuzzy set theory and their precision with repetitive identification.

4.2 Experiment Procedures

Stimuli: We adopted seven verbal expressions of tallness, like as those used in section 3. The first stimulus was used for dummy, thus the total six verbal expressions except for the first were analyzed.

Experiment design: This experiment consists of two sub experiments: "*non repetition experiment*" and "*repetition experiment*." In the non repetition experiment 29 subjects identified the seven stimuli using both the BASE method and the FGRS. We adopted the expression of both of the elements and their ratings as its user interface of the BASE method referring to the result obtained in the section 3. Meanwhile, we adopted a method for answering each two ends of the 1-level set and the support set as the FGRS [11]. These four boundary elements in the FGRS are measured in one pixel. We obtained the answering time and the ternary rating value per element in the BASE method, and the answering time and four boundary elements per stimulus in both. To examine influence of experiment order of both methods on results, we divided the subjects into two groups, and assigned the different order to each group. After identifying both of them the subjects were asked to compare ease of rate between the two methods using the pair comparison.

Meanwhile, in the repetition experiment ten subjects identified the same stimuli as in the non repetition experiment at four times. Five subjects from each group in the non repetition experiment participated. All the subjects identified in order from the BASE method to the FGRS at twice, and in opposite order at twice by turns. The results at the first repetition were diverted from the results in the non repetition experiment. There is no change of experiment design from the non repetition experiment except for that subjective evaluation of usability was executed after only first and forth repetition.

Subjects: All the subjects were graduate students and undergraduates at Kyoto Institute of Technology, and participated as volunteer. They were native Japanese speakers. So all the materials for experiment were displayed in Japanese. All the subjects had little knowledge on fuzzy set theory.

4.3 Results and Discussions

4.3.1 Usability of Both of the Methods: FGRS and BASE

Subjective evaluation of usability of FGRS and BASE: Figure 4.1 shows the results obtained from the pair comparison between both methods. As seen from the results of the non repetition experiment, those who have little knowledge on fuzzy set theory tend to prefer the BASE method to the FGRS in subjective usability. However, the difference of the frequencies between both methods (BASE : FGRS = 19 : 8) is not significant at 5% level (in two-tail) in the pair test. Meanwhile, in the repetition experiment the ratio of the BASE method versus the FGRS changed from 8 : 2 after the first repetition into 5 : 5 after the last repetition. As a reason of this change, we can think three subjects who change their evaluation come to imagine the whole shapes of the MFs easily, thanks to the repetition. So they regard the criterion of usability as "*smallness of effort spent on identification*" instead of "*easy to rate.*"

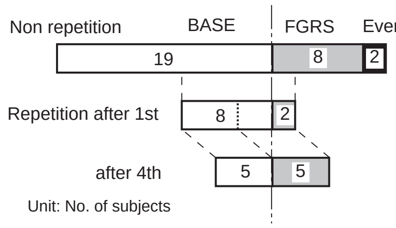


Figure 4.1. Results of pair comparison between the FGRS and the BASE method.

Usability based on answering time: Figure 4.2 shows the averaged answering time per hedge in the FGRS and the averaged answering time per element and the total number of ratings in the BASE method. In the non repetition experiment the averaged answering time per hedge in the BASE method and the FGRS took 42.15 *s* and 16.67 *s* respectively. The averaged answering time from the BASE method is about 2.5 times larger than that from the FGRS. Meanwhile, in the repetition experiment the answering time per hedge in the FGRS shortened as the experiment progressed as seen from the left figure in the figure 4.2. There are significant differences of the averaged answering time between the fourth repetition and the others at significance level of 0.5, 5 and 10% to the first, second and third repetition, in order, in the t-test of difference between mean values. It can be considered

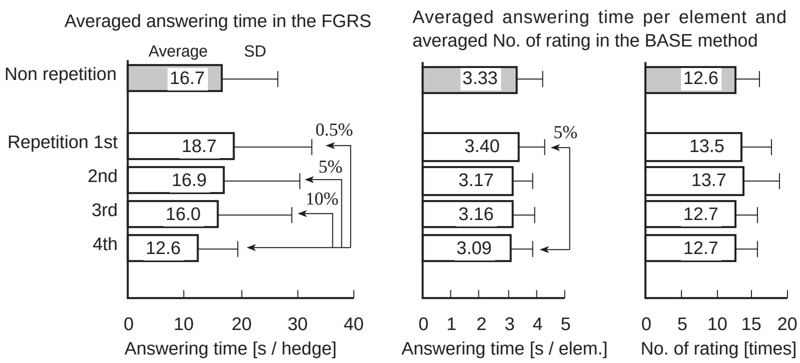


Figure 4.2. Results related to answering time from comparison between the FGRS and the Base method.

that the shortening the answering time origins from so-called "*learning effect*" related to shape of the MFs. Contrary in the BASE method the repetition of identification does not influence the averaged answering time per element, except for the pair of the first and fourth repetition (significance at 5% level). These results clearly show that the BASE method allows users to identify MFs without preceding training.

Conclusion of usability: In summary the results obtained are as follows; from subjective evaluation it is clear that those who have little previous knowledge on fuzzy set theory prefer the BASE method to the FGRS. Similarly the results of the answering time show that the BASE method does not require its user to learn the shape of MF. Consequently, it is concluded that the BASE method is satisfied with the aim of proposing the usable method without the knowledge on fuzzy set theory.

4.3.2 MFs Obtained from Both FGRS and BASE Methods

Evaluation measure: Now we discuss the MFs obtained from the BASE method and the FGRS. To compare the MF, we brought along following measures: *matching measure*, *similarity measure*, *position index*, *shape index*, *inclusion measure*. We adopted the measures related to the logical product, the logical sum and the cardinality as the matching, similarity and inclusion measures [12]. Formula (1) to (3) represents these measures.

Matching measure: $\mathrm{hgt}\ (A \cap B)$ (1)

Similarity measure: $|A \cap B| / |A \cup B|$ (2)

Inclusion measure ($A \supset B$): $|B| / |A \cup B|$ (3)

where A and B mean a fuzzy subset on a universe of discourse: X. $A \cap B$ and $A \cup B$ mean the logical product set and logical sum set between A and B, and $|A|$ denotes the cardinality of the fuzzy subset A. hgt (A) means maximum grade of the fuzzy subset A. The position index is the distance between two gravity centers of fuzzy sets. The shape index is the similarity measure of the two fuzzy sets that are shifted to meet their gravity centers. These indices can adequately describe a disagreement between two fuzzy sets in each meaning than the similarity measure [13].

Preparation of analysis: First, we calculated these measures between the MFs obtained from the BASE method and the FGRS for each same hedge and each subject, and then averaged them for each group. There were 70 and 68 effective data averaged in the group of order from the BASE method to the FGRS: B to F, and from the FGRS to the BASE

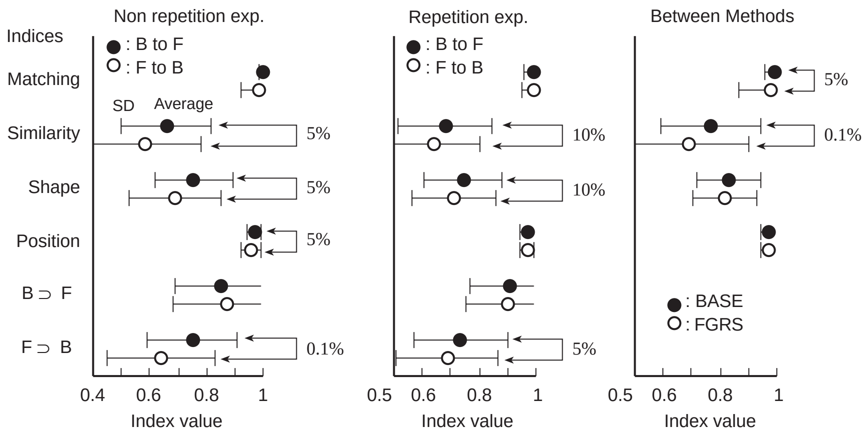


Figure 4.3. Results related to MF obtained from the FGRS and the BASE method

method: F to B respectively in the non repetition experiment. Meanwhile, in the repetition experiment 120 effective data were averaged for each group. **Figure 4.3** shows the results.

Difference between methods: The mean values for all subjects are 0.724 (SD: 0.153), 0.963 (0.032), 0.728 (0.144), and 0.971 (0.026) in order of the shape and the position index in the non-repetition, and the shape and the position index in the repetition. These values reveal the MFs obtained from both methods do not completely agree with each other. However, we must notice the difference of the minimum measurable interval between both methods. The interval in the FGRS is one pixel, while that in the BASE method is eight pixels. To estimate an influence of the difference, we think one trapezoidal fuzzy set averaged for all the sets obtained from this experiment. The widths of the 1-level set and the support set for the averaged fuzzy set are 0.138 and 0.259, normalized length of universe of discourse as unity. Meanwhile, the minimal interval in the BASE method corresponds to 0.031. Under these conditions, the position index calculated between the averaged fuzzy set and that shifted to 0.031 becomes 0.969. Similarly the shape index calculated between the original and that expanded each boundary of the original at 0.031 becomes 0.759. These values approximate to the obtained values, 0.963 and 0.724 respectively. Consequently the difference of the minimal measurable interval can decrease values of the measures, thus it is hard to remove the influence of the difference under these experimental conditions.

Influence of experiment order: As seen from the figure 4.3, the mean values for the “B to F” are higher than those from the “F to B” in both the non-repetition and the repetition experiment. Especially, for the similarity measure and the shape index, the differences between the averages are significant at 5% in the non-repetition experiment and 10% in the repetition experiment. These results suggest that identification using the BASE method prior to the FGRS helps the subjects to image the adequate shapes of the MFs and answer easily. That is the evidence of the BASE method being usable.

Stability of MF with repetition: We calculated the matching and similarity measure and the shape and position index of all possible combinations of four repetitions for each hedge and each subject, and averaged them for each method. The right figure in the figure 4.3 shows the results. As seen from the figure, for all four indices the values of the BASE method are higher than those of the FGRS. Moreover, there are significant differences of the matching and similarity measure at significance level of 5 and 0.1% in two-tail in the t-test of difference between averages. So these results reveal that the BASE method is superior to the FGRS in precision with repetitive identification.

Precision of MF: In summary the results obtained are as follows; the MFs obtained from both methods, FGRS and BASE, indicate good agreement except for irremovable difference on these experiment conditions. From the influence of experiment order on the MFs the BASE method shows higher usability than the FGRS. Moreover, the BASE method turns out to be superior to the FGRS in stability in the experiment of repetitive identification of MFs. Consequently, we conclude the BASE method is not only usable, but also superior to the FGRS in precision.

5. Conclusions

We have proposed the new method for identifying a trapezoidal MF, named the boundary asymptotic estimation method (BASE). The features of the method are the ternary rating of membership grades, asymptotic estimation of boundaries of 1-level set and support set, and effective and recursive selection of elements using computers. The results of verification experiment have shown that the BASE method is superior to the computerized fuzzy graphic rating scale both in usability for "*novice*" users of fuzzy set theory and precision.

We have already rewritten the program of the BASE method in the JAVA language. The BASE method therefore is available for use with internet browsers and works along with other systems on any operating system. In fact, Yoshikawa has already used the BASE method for obtaining individual meaning of verbal hedges in computer aided design of categorical rating scales [14].

However, the problem of forced binary rating has remained. To fix and reduce it, I would like to find and test other measures adequate for detecting confusion of rating during identification.

Acknowledgments

This study was supported by Tateishi Science and Technology Foundation in 1994.

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