Intermediate Expressions between Fuzzy Concepts Corresponding to Subjective Judgment

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Abstract

This paper aims to examine three operators for calculating an intermediate expression between fuzzy concepts; the Between-set: Betw, normalized logical product set: NLP, and type-2 average set: T2A, through psychological experiments. In the experiments, 22 subjects identify seven primary verbal expressions of tallness and 15 intermediate expressions between two of the seven primary expressions as fuzzy sets. Then the intermediate fuzzy sets identified directly by the subjects, IFSs, are compared with three different fuzzy sets calculated from pairs of the primary fuzzy sets by using the three operators.

The results obtained are as follows. 1) In the three operators adopted in this study, the T2A corresponds to the IFS most. 2) In the sense of estimating gravity center of the IFSs, however, there is no significant difference between the Betw and the T2A.

1 Introduction

Meaning of intermediate expression: One of the functions of word as label is to categorize a continuous universe of discourse. Narrowing the area covered by one word is necessary to describe the universe of discourse close. It can be said that a transitional area between two words, whose elements have low typicality for both words, obtains a new label. The area implies an intermediate area between the two labels. Namely, the area is expressed as "*the area between labels of A and B*" until it owns one specified name.

Importance in application: Suppose that we utilize an operator for making intermediate expressions, such as a fuzzy set operation. Then, for example, we expect to improve performance of systems whose inputs and outputs are expressed as fuzzy set by introducing intermediate expressions to describe closely. We here need a new fuzzy set operator that calculates the intermediate expression. The operator needs to adequately formulate a degree of intermediate which she/he subjectively estimates based on two fuzzy concepts. For the purpose, we have to clear correspondence between calculated intermediate expression and one estimated subjectively through experiment.

Previous studies: *Kay and McDaniel* (K&M) [1], and *Yabuuchi* [2] formulated a process that categories of the derived colors, such as orange and purple, combined from categories of the primary colors, such as red, blue and yellow, as fuzzy set operators. They defined a fuzzy subset (FSS) of the derived color as logical product set (LPS) between two FSSs of the primary colors. They multiplied the LPS into 2 in scalar to normalize, supposing that the FSSs of the primary colors are orthogonal.

Meanwhile, *Yoshikawa* [3] proposed the Between-set (Betw) to improve ability for categorizing a universe of discourse using extent adverbs. The Betw is derived from absolute difference operator of membership degrees. He has already used the operator to calculate psychological scale values of intermediate categories, such as "*the category between very tall and fairly tall*," in categorical rating scales, and validated usage of the Betw [4].

Problems of the previous studies: *K&M* aimed to formulate the intermediate expression of the particular objects, that is, categories of colors. In other words, the orthogonality among FSSs makes hard to extend the operator into objects whose the orthogonality is not content [3]. Meanwhile, the Betw was defined in consideration of general applications. Although introducing intermediate expressions was validated, it was insufficient to compare the Betw with other operators through psychological experiment. Furthermore, other operators may possibly formulate intermediate degree more than enough proposed.

Aim of this study: This study aims to propose fuzzy set operators that well formulate intermediate degree which one can suppose from two fuzzy concepts. In this paper, we examine the following three operators; *average set in type-2 fuzzy set*, *normalized logical product set (NLP) derived by K&M* and *Yoshikawa's Betw*. Namely, we compare intermediate fuzzy sets that subjects identify directly (hereafter, identified intermediate fuzzy set: IFS) with ones that are calculated from the two fuzzy sets by the operators (hereafter, calculated intermediate fuzzy set: CFS).

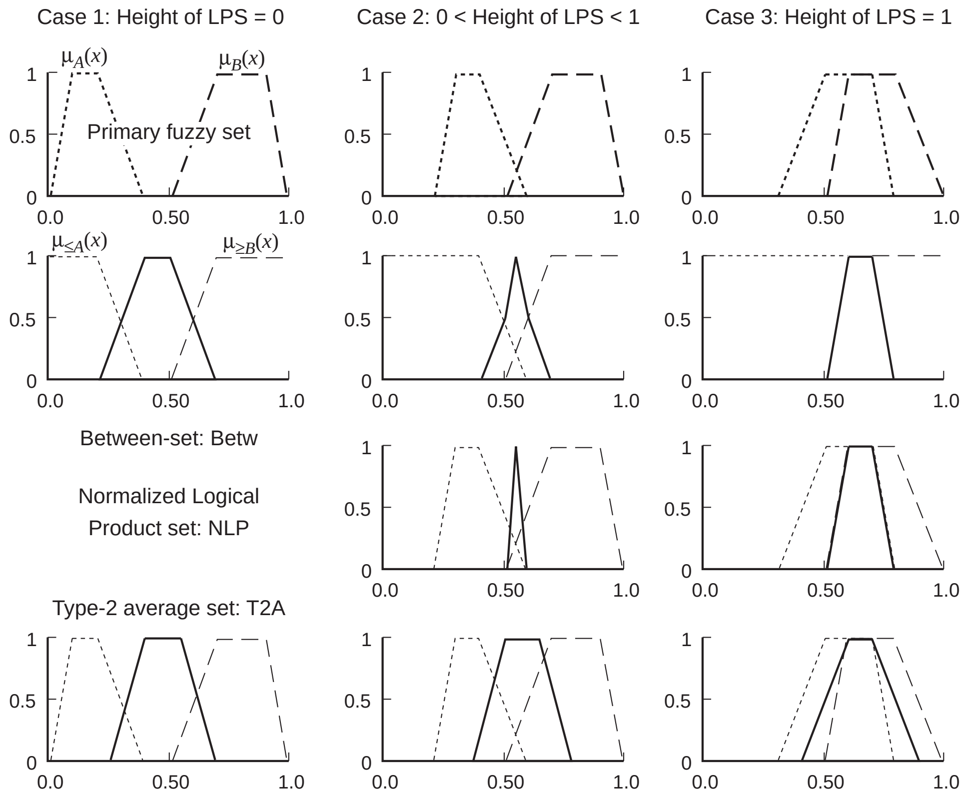

Figure 1: Illustration of three operators for calculating intermediate fuzzy sets and influence of relative position between primary fuzzy sets on those calculated intermediate fuzzy sets.

2 Intermediate operations adopted

2.1 The Between-set: Betw

Yoshikawa defined a state of intermediate as a state where one cannot clearly describe which categories objects belong to. He formulated the state as complement of absolute difference set between two FSSs of *A* and *B* by equation (1) [3]. Where *A* is equal to or less than *B* ($A \leq B$).

$$\mu_{Betw}(x) = 1 - | \mu_{\leq A}(x) - \mu_{\geq B}(x) | \qquad (1)$$

However, membership function (MF) of the Betw becomes more than zero in inadequate area, calculating the Betw from *A* and *B* directly. To avoid this problem, *A* and *B* are replaced by two FSSs named "*less than A* ($\leq A$)" and "*more than B* ($\geq B$)" respectively as shown in **Figure 1**. Moreover, *Yoshikawa* clarified some mathematical properties of the Betw.

2.2 Normalized logical product set: NLP

This operator originates from complement of absolute difference set, as same as the Betw. *K&M* supposed orthogonality between *A* and *B*, and derived LPS multiplied into 2 in scalar, as shown in equation (2) [1].

$$\mu_{2LP}(x) = 2 \cdot \min\{\mu_A(x), \mu_B(x)\} \qquad (2)$$

In this study, however, we do not condition on orthogonality among FSSs identified. In some cases maximum membership grade of CFS does not reach unity. We, therefore, consider derivation into the LPS in *K&M*, and interpret "*multiplying into 2 in scalar*" as normalization as shown in equation (3). Logically the NLP becomes empty set under the condition where the LPS calculated from *A* and *B* is empty set in figure 1.

$$\mu_{NLP}(x) = \frac{\min\{\mu_A(x), \mu_B(x)\}}{\sup_{x \in X} \{\min\{\mu_A(x), \mu_B(x)\}\}} \qquad (3)$$

2.3 Type-2 average set: T2A

Type 2 average set (T2A) is defined as arithmetical mean between elements of infimum or supremum for all level sets of *A* and *B*, as shown in equation (4). Namely, the T2A is the average operator between two fuzzy numbers based on the extension principle.

$$\mu_{T2A}(x) = \sup_{x = (m+n)/2} \{\min\{\mu_A(m), \mu_B(n)\}\} \qquad (4)$$

3 Experimental methods and conditions

Stimuli: In this study, two kinds of stimuli were used; *primary expressions* and *intermediate expressions*, respectively. The primary expressions were composed seven verbal hedges (namely extent adverbs) and "*tall*," such as "*very tall*." Meanwhile, 15 intermediate expressions consisted of two of the seven primary expressions, such as "*tallness between very tall and fairly tall*."

Methods: The experiment consists of two parts. In the first part, subjects identified seven MFs for the primary expressions. Then they identified 15 MFs for the intermediate expressions in the second part. At the beginning of both parts, the subjects were instructed to imagine a situation where you express tallness of Japanese males using extent adverbs. There was no limitation of answering time in both parts. The experiment was carried out on web page using internet browser. We adopted the BASE method [5] in JAVA edition as MF identification method. Therefore, the MFs obtained were either trapezoidal or triangular type.

Subjects: The subjects were 21 students at Kyoto Institute of Technology and one participant from outside the institute. All of them were Japanese native speakers, accordingly all materials used in the experiments were displayed in Japanese. They voluntarily participated in and had little technical knowledge on fuzzy set theory. Six subjects were excluded from following analysis, since the MFs identified by them included problems. After all, number of effective subjects were sixteen.

4 Results and discussions

Evaluation indices: In order to evaluate degree of matching between MFs of the IFSs and the CFSs, we need some evaluation indices. The indices have to be capable of evaluating subjective degree of matching between the fuzzy concepts. Referred to the results on matching measures corresponding to subjective judgment of matching [6], we adopted the following eight indices; *the shape index SI*(*A*,*B*) and *the position index PI*(*A*,*B*) [7], and *maximum value* (suffix: ∞), *integral value* (1) and *value at 1 level set* (*) for *the Dissemblance index Δ*(*A*,*B*) and *the Housdorff metric q*(*A*,*B*) respectively [8]. The definitions of their indices are shown in equation (5) through (12). Refer to reference for each detail.

$$gc(A) = \int x \cdot \mu_A(x)\, dx \Big/ \int \mu_A(x)\, dx$$

$$SI(A,B) = \frac{\int \min\{\mu_A(x'),\mu_B(x)\}\, dx}{\int \max\{\mu_A(x'),\mu_B(x)\}\, dx} \quad (5)$$

$$\text{where } x' = x - gc(A) + gc(B)$$

$$PI(A,B) = 1 - |gc(A) - gc(B)| \quad (6)$$

$$q(A,B) = \max\{|a_L - b_L|, |a_U - b_U|\}$$

$$q_\infty(A,B) = \sup_{\alpha \geq 0} q(A_\alpha,B_\alpha) \quad (7)$$

$$q_1(A,B) = \int_{\alpha=0}^{1} q(A_\alpha,B_\alpha)\, d\alpha \quad (8)$$

$$q_*(A,B) = q(A_{1.0},B_{1.0}) \quad (9)$$

$$\Delta(A,B) = (|a_L - b_L| + |a_U - b_U|) / 2(\beta_U - \beta_L)$$

$$\Delta_\infty(A,B) = \sup_{\alpha \geq 0} \Delta(A_\alpha,B_\alpha) \quad (10)$$

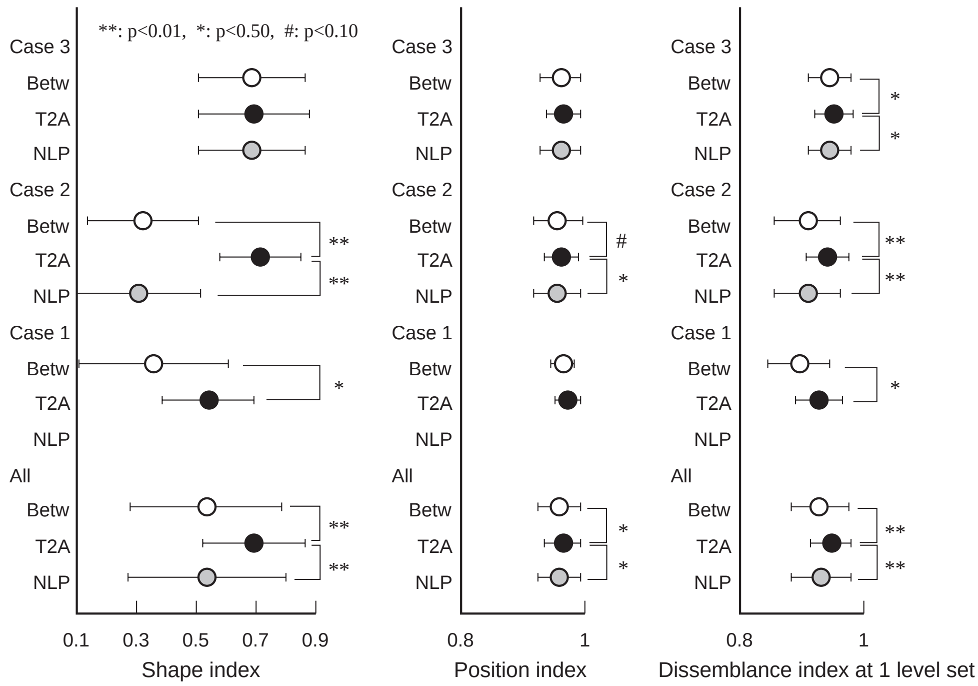

Figure 2: Average values of the indices for each case and results on t-test between operators

$$\Delta_1(A,B) = \int_{\alpha=0}^{1} \Delta(A_\alpha, B_\alpha)\, d\alpha \quad (11)$$

$$\Delta_*(A,B) = \Delta(A_{1.0}, B_{1.0}) \quad (12)$$

Where gc(A) denotes gravity center of FSS of A, a_L and a_U elements of infimum and supremum of crisp set of A, and A_α α level set of FSS of A, respectively.

Δ and q work as disagreement measures. To convert into matching measures, we first normalize diameter of the universe of discourse to unity, then subtract their normalized values of the six indices from unity. Therefore, for all indices, the closer the values become unity, the more MFs of CFS correspond to ones of IFS.

Discussions on all data: First, the eight indices were calculated between MF of IFS that each subject identifies and MFs of CFS by the three operators for each stimulus and each subject. Then, their mean values and their SDs for each index were calculated from 240 data, namely 15 stimuli by 16 subjects. **Figure 2** shows results of *SI*, *PI* and Δ_* only due to limitation of space.

As seen from the mean values, the values for the T2A are closer to unity than the other operators for all of the eight indices. Moreover, there are significant differences of the mean values between the T2A and the others at significance level of 1% for the seven indices except for *PI*. Consequently, the T2A corresponds to IFS most among the three operators used in this study.

Influence of position between primary FSSs: Next, we examine influence of position between primary FSSs used for calculating CFSs on index values. In this study, we classify three cases based on height of LPS calculated from two primary FSSs, as shown in figure 1. Then, we name the height of zero, unity and otherwise case 1, 3, and 2, respectively.

In this experiment, the data in the case 1 are 13 examples in all of 240 examples. Obviously, we cannot calculate the CFSs for these data using the NLP. As *Yoshikawa* has pointed out in the reference [3], the NLP is not suitable for the intermediate operator due to this limitation on its application.

In the case 3, there is no significant difference of the mean values among the three operators at significance level of 10%, except for Δ_*. Contrary, in the case 2, there are significant differences of the mean values between the T2A and the others at 1% level except for *PI*, as seen from figure 2. In the case 2, the 1 level sets of the CFSs by the Betw and the NLP become one point, as shown in figure 1. On the other hand, all of the IFS obtained in this experiment are trapezoidal MFs, although the BASE method could give triangular MF. Consequently, it can be thought that the difference from shape of MF causes values of Δ and q calculated from boundaries of level sets and *SI* to decrease.

Comparison among indices: As limited to *PI*, related to distance between gravity centers of primary FSSs, the difference of the mean values for all data is small among the three operators, especially between the Betw and the T2A. Therefore, the Betw is equivalent to the T2A in the sense of describing gravity centers of the IFS.

The weight of membership degree is not adequately considered in Δ_∞, Δ_1, q_∞ and q_1. In some cases, their values are dominated by the level sets closer to zero. We need to be attentive when we use these indices for evaluating matching degree.

5 Conclusion

In this paper, we have compared the three operators to propose the most suitable operator corresponding to the intermediate expression directly identified by subjects. The results obtained have shown that *the type-2 average set* is most preferable among the three operators adopted in this study. Another finding is that *the Between-set* is equivalent to the T2A in the matter of describing gravity centers of the intermediate FSSs. Same equivalency is seen in the case 3 where the LPS for 1 level set of the primary FSSs exists.

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