Verification of Intermediate Operators between Fuzzy Sets Corresponding to Subjective Judgment of Intermediate

Ayumi YOSHIKAWA

Faculty of Education
Okayama University
ayumi@sip.ed.okayama-u.ac.jp

Abstract

Aims of this paper are to propose and verify suitable operators for calculating intermediate fuzzy set corresponding to subjective judgment of intermediate between two fuzzy concepts. Three operators for calculating intermediate fuzzy set: Normalized logical product set (NLP), the Between-set (Betw), and Type-2 average set (T2A), are compared through experiments. The results obtained are as follows: 1) The NLP is inferior to the others in coverage. 2) The Betw corresponds to the T2A in estimating gravity centers of identified intermediate fuzzy sets (IIFS). 3) There is little influence of the differences of the universal sets upon the correspondence. 4) Individual differences exist in the correspondence of these operators with the IIFSs.

1. Introduction

Background of this study: People often use verbal phrases, such as "*very tall*" and so on, to express, categorize and communicate subjective degree of a fuzzy concept in daily life. Since fuzzy sets for these phrases are sparse on a universal set, some elements on it could have low membership grades for those fuzzy sets. In such case people cannot find one suitable phrase to express the degree of the element. Thus, an intermediate expression, such as "*degree between very tall and fairly tall*" is often substituted with one word. Kay and McDaniel (K&M) [1] and Yabuuchi [2] formulated a process that a derived color is combined from two primary colors as fuzzy set operators, e.g., a fuzzy set for "*orange*" is combined from fuzzy sets for "*yellow*" and "*red*" with logical product operator and normalization. Yoshikawa also proposed the Between-set as an intermediate operator and showed some mathematical properties of it [3].

Importance of this study: Applications of fuzzy set theory to real world, especially a user interface in a human-computer interactive system, utilize verbal labels to approximate degree of elements. In those systems, an intermediate expression between two fuzzy concepts is useful to improve the performance of the systems at translation process of the degree of object into verbal expression. On this occasion, an intermediate fuzzy set calculated from different fuzzy sets with an intermediate operator must correspond to a fuzzy set that people directly identify for the intermediate expression. Therefore, the topic is important not only from an aspect of cognitive psychology but also from an aspect of computer science.

Previous studies related: In view of the above importance, Yoshikawa [4] has already studied on this topic. He compared the following three intermediate operators based on correspondence of them with intermediate fuzzy sets directly identified by subjects through experiments: the Between set (Betw), normalized logical product set (NLP), and type-2 average set (T2A). The results obtained revealed that the T2A, as intermediate operator, has better capability than the others overall. However, we need to verify these results through experiment to make them more reliable and valuable.

Three aims of this study: The first aim of this paper is to exemplify correspondence of above three operators with subjective judgment of intermediate in order to review the Yoshikawa's results [4]. Another aim is clarifying to what degree differences of the universal sets for fuzzy sets influence the correspondence. The other aim is to clarify individual differences of the correspondence. Thus, the following two different intermediate fuzzy sets for same intermediate were compared through experiments: One is an identified intermediate fuzzy set (IIFS), directly identified by subjects. The other is a calculated intermediate fuzzy set (CIFS), calculated from two primary fuzzy sets with the three operators.

2. Intermediate Operators Adopted

In this chapter, I will show definitions of three intermediate operators adopted in this study, also illustrate relationships between shape of the CIFSs and relative position of primary fuzzy sets.

2.1 The Between-set: Betw

Definition: Yoshikawa defined a state of intermediate as a state that an object's belonging category couldn't be clearly described. He formulated the state as complement of absolute difference set between two normal convex fuzzy subsets (FSSs) of *A* and *B* by equation (1) [3]. Where *A* is equal to or less than *B* ($A \leq B$).

$$\mu_{Betw}(x) = 1 - |\, \mu_{\leq A}(x) - \mu_{\geq B}(x) \,| \qquad (1)$$

However, membership function (MF) of the Betw becomes more than zero in inadequate area, calculating the Betw from *A* and *B* directly. To avoid this problem, *A* and *B* are replaced with two FSSs named "*less than A* ($\leq A$)" and "*more than B* ($\geq B$)" respectively as shown in **Figure 1**. The MFs obtained from the Betw are shown in Figure 1 every relative position of two primary FSSs.

Expansion of definition: The Betw is not defined on condition if *A* includes *B* or *B* includes *A*, namely, case 4. In this case, we use the NLP instead of the Betw. This expansion equals that the less than set and more than set are calculated at lower and upper side separately. For examples, if *B* includes *A*, then *B* is replaced with the universal set, namely, "*less than B*" and "*more than B*" as shown in Figure 1.

2.2 Normalized Logical Product Set: NLP

Definition: This operator originates from complement of absolute difference set, likewise the Betw. K&M supposed orthogonality between *A* and *B*, and derived logical product set (LPS) multiplied into 2 in scalar, as shown in equation (2) [1].

$$\mu_{2LP}(x) = 2 \cdot \min \{\mu_A(x), \mu_B(x)\} \qquad (2)$$

In this study, however, we do not condition on orthogonality among FSSs identified. In some cases, maximum membership grade of CIFS does not reach unity. We, therefore, interpret "*multiplying into 2 in scalar*" as normalization as shown in equation (3), considering the derivation into the LPS in K&M. The NLP becomes empty set iff the LPS calculated from *A* and *B* is empty set in figure 1.

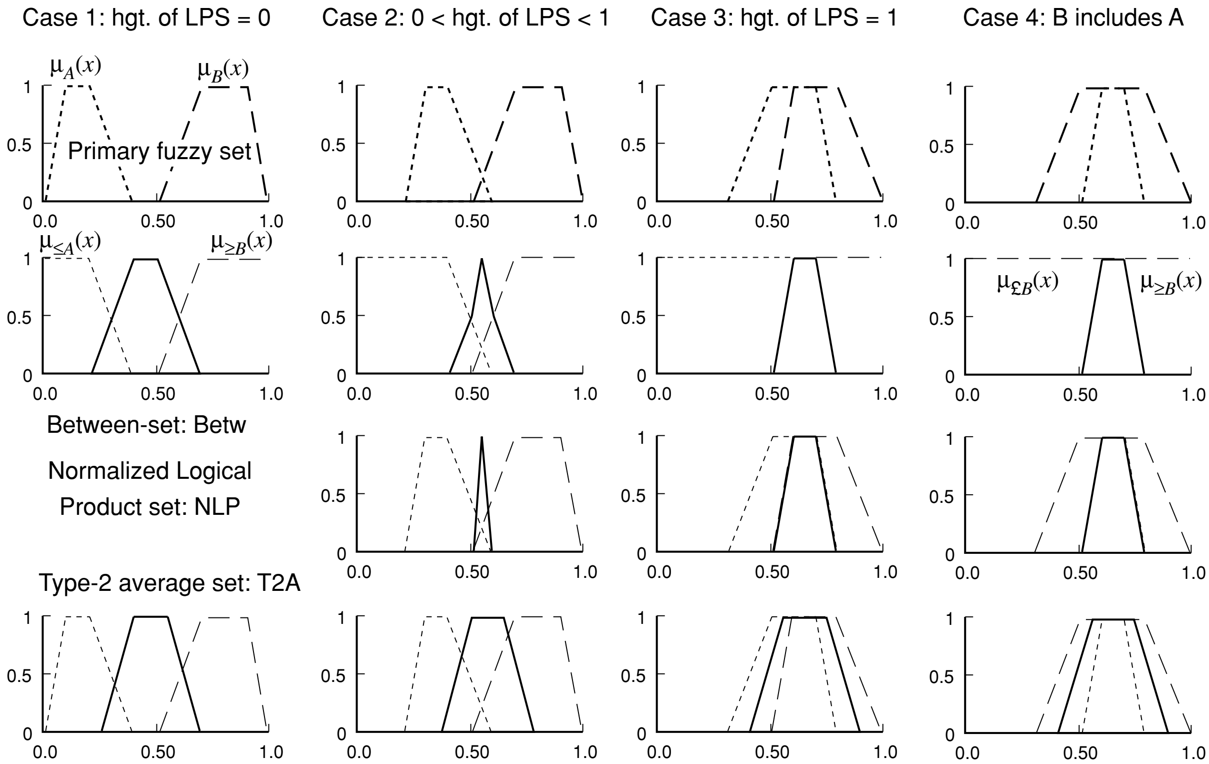

Figure 1: Illustration of three operators for calculating intermediate fuzzy sets and influence of relative position between primary fuzzy sets on those calculated intermediate fuzzy sets.

$$\mu_{NLP}(x) = \frac{\min\{\mu_A(x), \mu_B(x)\}}{\sup_{x \in X}\{\min\{\mu_A(x), \mu_B(x)\}\}} \quad (3)$$

2.3 Type-2 Average Set: T2A

Definition: T2A is defined as arithmetical mean between two elements of infimum for all level sets of A and B and that of supremum, as shown in equation (4). In other words, this is the average operator between two fuzzy numbers based on the extension principle.

$$\mu_{T2A}(x) = \sup_{x = (m+n)/2}\{\min\{\mu_A(m), \mu_B(n)\}\} \quad (4)$$

3. Experimental Methods and Conditions

Experimental methods: This experiment consists of four sub tasks. In the task 1 and 3, subjects identified MFs for seven phrases of tallness. These phrases of tallness consist of seven verbal hedges and word of tall, such as "*very tall*." On the other hand, in the task 2 and 4, they identified MFs for 21 intermediate phrases of it. The intermediate phrases were combined from all possible pairs of above seven phrases, such as "*tallness between very tall and fairly tall*." The presentation orders of the phrases in the task 1 and 3 are identical, and those in the task 2 and 4 are identical.

The fuzzy graphic rating scale method was adopted as the identification method. In the task 1 and 2, subjective degree of tallness was used as the universal set. In other words, both extreme point of the rating scale mean "*absolutely short*" and "*absolutely tall*" respectively. In the task 3 and 4, physical length ranged from 130 cm to 230 cm with 11 ticks of every 10 cm was used. The subjects marked four points that correspond to two intervals: One is that elements on the interval belong to the phrase most. The other is that elements on the interval belong to the phrase at least. **Figure 2** shows examples of both scales for the task 1 and 3. **Table 1** summarizes relationships among the four tasks.

The four tasks were executed in same order from the task 1 through 4 for all subjects. A brochure that contains instructions and answer sheets for all tasks was used. The brochures were distributed, and gathered after the subjects finished to answer. No time limit was set to answer.

Table 1: Relationships among the four tasks

Universal set	Phrases: Primary	Phrases: Intermediate
Subjective	Task 1	Task 2
Physical	Task 3	Task 4

Subjects: Thirty-nine students at Okayama University participated in this experiment as volunteer. They were all Japanese native speakers, and thus the brochures used were printed in Japanese.

4. Results and discussions

4.1 Preprocessing

Constructing MFs: First, trapezoidal or triangular MF was constructed from four or three points that the subject marked on rating scale for each primary and intermediate phrase. Next CIFSs were computed with the three operators from all possible pairs of the MFs for the primary phrases. Then the three of CIFSs were compared with the IIFS for same intermediate phrase in order to examine correspondence among the three operators with subjective judgment of intermediate. Moreover the CIFSs were classified into the following four cases shown in Figure 1 according to height of LPS between two primary fuzzy sets: in case 1 the height is equal to zero, in case 2 it ranges zero through unity, in case 3 it is equal to unity, and in case 4 one 1-level set includes another.

Evaluation indices of correspondence: To evaluate the correspondence we need some evaluation indices that show to what degree two fuzzy sets correspond each other. The indices have to be compatible with subjective judgment of matching between fuzzy concepts. Therefore the following eight indices were adopted referring to literature [5]: the shape index $SI(A,B)$, and the position index $PI(A,B)$ [6], and maximum value, integral value and value at 1 level set (suffix: "∞",

(a) universal set: subjective tallness (task 1)

(b) universal set: physical height (task 3)

Figure 2: Examples of fuzzy graphic rating scales used in the experiments.

"1", "*" in order) for both the Dissemblance index $\Delta(A, B)$ and the Housdorff metric $q(A, B)$ [7]. Their definitions are shown in equation (5) through (12).

$$SI(A,B) = \| \mu_A(x') \wedge \mu_B(x) \| / \| \mu_A(x') \vee \mu_B(x) \| \quad (5)$$

$$PI(A,B) = 1 - | gc(A) - gc(B) | \quad (6)$$

$$q(A,B) = \max \{ | a_L - b_L |, | a_U - b_U | \}$$

$$q_\infty(A,B) = \sup_{\alpha \geq 0} q(A_\alpha, B_\alpha) \quad (7)$$

$$q_1(A,B) = \int_{\alpha=0}^{1} q(A_\alpha, B_\alpha)\, d\alpha \quad (8)$$

$$q_*(A,B) = q(A_{1.0}, B_{1.0}) \quad (9)$$

$$\Delta(A,B) = (| a_L - b_L | + | a_U - b_U |) / 2(\beta_U - \beta_L)$$

$$\Delta_\infty(A,B) = \sup_{\alpha \geq 0} \Delta(A_\alpha, B_\alpha) \quad (10)$$

$$\Delta_1(A,B) = \int_{\alpha=0}^{1} \Delta(A_\alpha, B_\alpha)\, d\alpha \quad (11)$$

$$\Delta_*(A,B) = \Delta(A_{1.0}, B_{1.0}) \quad (12)$$

Where X is the universal set [0, 1], and A, B are FSSs on X. x belongs to X. $\|A\|$ denotes cardinality of A. $gc(A)$, $gc(B)$ are gravity center of A, B. x' is $x\text{-}gc(A)+gc(B)$. A_α, B_α denote α-level sets for A, B. a_L, b_L and a_U, b_U are infimum and supremum respectively. Note that both $\Delta(A,B)$ and $q(A,B)$ show incompatibility between two FSSs. Thus we subtract those values from unity, and redefine the translated values as evaluation indices.

4.2 Comparison of Correspondence among Three Operators

Scope of this section: Aim of this section is reviewing Yoshikawa's results in [4] through this work. **Table 2** shows mean values, SDs over all data, and results of T-test between the mean values of four evaluation indices in the Betw and T2A. These values are summed up for each case, all, and each type of universal sets. The results obtained are as follows.

Flaw of the NLP: The NLP is inferior to the Betw and T2A in coverage. Table 2 shows that there are 527 data (32% of all data) classified into case 1. In this case the NLP cannot calculate intermediate fuzzy sets due to the definition as mentioned in chapter 2. Consequently this result clearly shows that its definition puts limitations on coverage of the NLP in real applications. Therefore I limit to examine the results of the Betw and T2A hereafter.

Equivalence of estimating gravity center: The *PI* tells how fuzzy sets differ in their gravity centers. As seen from the results of *PI* for all data in Table 2, there is no significant difference between both mean values at significance level of 10%. Consequently the Betw and T2A have same capability

Table2: Correspondence of calculated intermediate MFs with identified intermediate MFs

Conditions	No.	Shape Index: SI			Position Index: PI			Dissemblance: Δ·			Housdorff: q·		
Both	data	Betw	T2A	T	Betw	T2A	T	Betw	T2A	T	Betw	T2A	T
Case4	49	0.661	0.633		0.948	0.954		0.947	0.943		0.927	0.921	
		0.212	0.203		0.046	0.047		0.048	0.048		0.075	0.075	
Case3	436	0.604	0.679	1	0.960	0.962		0.951	0.954		0.931	0.937	10
		0.216	0.220		0.038	0.037		0.042	0.037		0.060	0.055	
Case2	626	0.377	0.681	1	0.953	0.953		0.935	0.943	1	0.910	0.927	1
		0.193	0.199		0.044	0.044		0.048	0.046		0.062	0.059	
Case1	527	0.576	0.615	1	0.959	0.957		0.932	0.930		0.907	0.913	10
		0.224	0.219		0.035	0.037		0.045	0.051		0.061	0.062	
All	1638	0.510	0.658	1	0.957	0.957		0.939	0.942	5	0.915	0.925	1
		0.235	0.213		0.040	0.040		0.046	0.047		0.063	0.060	
Subjective	819	0.516	0.654	1	0.958	0.961	5	0.936	0.943	1	0.912	0.926	1
tallness		0.234	0.215		0.035	0.033		0.042	0.042		0.057	0.052	
Physical	819	0.504	0.661	1	0.956	0.952	10	0.941	0.941		0.918	0.923	10
height		0.237	0.211		0.044	0.046		0.049	0.051		0.067	0.067	

T: t-test of differences between mean values, 1: $p<0.01$, 5: $p<0.05$, 10: $p<0.10$.

to estimate gravity centers of IIFSs. Moreover the result that both of the mean values reach 0.957 shows that they can estimate the gravity centers of IIFSs about 4% of shift in diameter of the universal set. In other words, the difference corresponds to about 4 mm under this experimental condition; the diameter of the rating scale is at 100 mm in this work.

Differences between Betw and T2A: The results from seven evaluation indices for all data, except the *PI*, show that the T2A is superior to the Betw and there are significant differences between the mean values at significance level of 5%. Especially as the difference in the *SI* becomes larger, the T2A becomes better in graphical view of what an operator estimates both in the gravity centers of IIFSs and the shape of them.

Differences among the four cases: As seen from the result for each case, seven indices except the *PI* reveal significant differences between the mean values at significance level of 1% in the case 2, namely a height of the LPS between two primary fuzzy sets ranges from more than zero to less than unity. As Yoshikawa pointed out [4] , that is the reason the CIFSs with the Betw are triangular MFs, but the CIFSs with the T2A and almost IIFSs in fact are trapezoidal MFs. This discrepancy among the shape of them is found in this study, since our subjects also more prefer to identify as trapezoidal MFs rather than triangular MFs in every case.

Brief summaries: As said at the top of this section, this section aims to review the Yoshikawa's results in [4]. So this experiment was executed in the same way for different subject battery. Accordingly, the four results shown above are consistent with them, and validate them. Hence, it is an evidence that these results are reliable.

4.3 Influence of Difference of Universal Sets

Scope of this section: In this section, influence of difference of universal sets upon correspondence between CIFSs and IIFSs is examined. Yoshikawa's original study treated only a condition that the universal set is subjective tallness, namely, subjective degree. So, it is unclear that differences of universal sets do not influence the correspondence of these operators with the IIFSs. Thus, as mentioned in Chapter 3, the subjects were asked to do same tasks with two different universal sets, that is, subjective tallness (task 1, 3) and physical height (task 2, 4) , then the two results obtained are going to be compared in order to clear above topic.

Little influence of difference of universal sets: Two bottom rows in Table 2 show the results obtained from two different universal sets. As seen from the two rows, the differences of the index values between the Betw and T2A obtained from subjective tallness are equal to those obtained from physical height. However, there is a tendency that the differences in physical height are smaller, compared to those in subjective tallness. Consequently, it can be concluded that there is little influence of the differences of the universal sets upon the correspondence based on these results.

4.4 Individual differences of Correspondence

Scope of this section: We have considered the correspondence based on the mean values over all subjects in Section 4.2 and 4.3. Now, we will examine individual differences of the correspondence. It is important to consider the individual differences because they are likely to exist in the intermediate operators, as they exist in MFs. First the differences of two mean values for each subject and each universal set were calculated, and then the frequencies of samples were summed for each inequality, that is, Betw<T2A and Betw>T2A.

Existence of individual differences: **Table 3** shows these results for the four indices, and the figures in parenthesis mean the number of significant difference between the mean values at significance level of 10%. As seen from Table 3, there are samples whose index values for the Betw are larger than the T2A, contrary to the mean values for all subjects. This is caused by the following two reasons: One is that relative location of their MFs for primary phrases is biased. The other is that the Betw estimates IIFSs more than the T2A in those samples. If we adopt the later as the reason for the individual differences, we must consider individual difference of adequate intermediate operators. However, we need more experiments to clarify the reason, and I will carry them out in future studies.

Table3: Individual differences

	SI	PI	$\Delta\cdot$	$q\cdot$
Betw<T2A	71	49	46	57
(p<0.10)	(50)	(9)	(15)	(29)
Betw>T2A	7	29	32	21
(p<0.10)	(3)	(5)	(5)	(6)

Unit: No. of samples.

5. Conclusion

In this paper, we have examined the following three topics through experiments: the first is to exemplify correspondence of the three intermediate operators with subjective judgment of intermediate in order to review the Yoshikawa's results, the second is to show to what degree differences of the universal sets influence the correspondence, and the third is to clarify individual differences of the correspondence. The results obtained are as follows: First, the NLP is inferior to the Betw and T2A in coverage originated from its definition. Then, the Betw corresponds to the T2A in estimating gravity centers of the IIFSs, but is inferior to T2A in estimating the shape of them. These results confirm the Yoshikawa's results. Second, there is little influence of the difference of the universal sets upon the correspondence. Lastly, individual differences exist in the correspondence of these operators with the IIFSs, and some samples show the Betw is superior to the T2A contrary to the mean values for all subjects.

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